OCR
Oxford Cambridge and RSA

AS Level Further Mathematics A

Y533/01 Mechanics

Tuesday 22 May 2018 – Afternoon

Time allowed: 1 hour 15 minutes

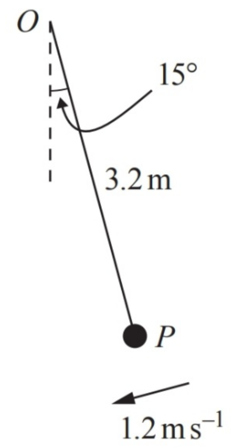
You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

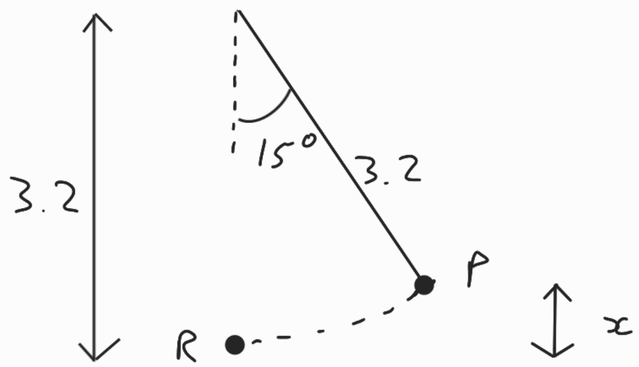
INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

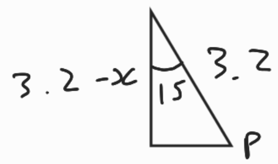
1



A particle P of mass m kg is attached to one end of a light inextensible string of length 3.2 m. The other end of the string is attached to a fixed point O. The particle is held at rest, with the string taut and making an angle of 15° with the vertical. It is then projected with velocity $1.2\,\mathrm{m\,s^{-1}}$ in a direction perpendicular to OP and with a downwards component so that it begins to move in a vertical circle (see diagram). In the ensuing motion the string remains taut and the angle it makes with the downwards vertical through O is denoted by $\theta°$.

(i) Find the speed of P at the point on its path vertically below O. **[4]**





$$\cos 15 = \frac{3.2 - x}{3.2}$$

$$x = 3.2 - 3.2\cos 15$$

$$x = 3.2(1 - \cos 15)$$

So at R, the particle has moved $3.2(1-\cos 15)$ m down from position P.

$$\Delta PE = mg\Delta h$$
$$= mg \times 3.2(1-\cos 15)$$
$$= 1.0685\ldots m$$

$$KE \text{ at } P = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2}m(1.2)^2$$
$$= 0.72m$$

$$KE \text{ at } R = \tfrac{1}{2}mv^2$$

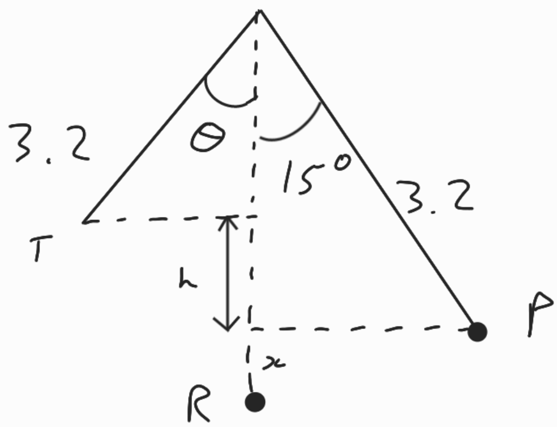
$\therefore$ By conservation of energy:

$$\tfrac{1}{2}\cancel{m}v^2 = 0.72\cancel{m} + 1.069\cancel{m}$$
$$\tfrac{1}{2}v^2 = 1.789$$
$$v^2 = 3.577$$
$$v = 1.89\ ms^{-1}$$

← +ve as same direction as velocity given as +ve in question.

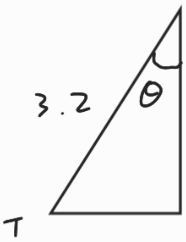
(ii) Find the value of θ at the point where P first comes to instantaneous rest. **[2]**

Let particle first come to rest at point T:



$$x = 3.2(1-\cos 15)$$
$$x = 0.109$$

$$KE \text{ at } T + \text{gained } PE \text{ from } P \to T = KE \text{ at } P$$
$$0 + \cancel{m}gh = 0.72\cancel{m}$$
$$h = \frac{0.72}{9.8} = 0.0734\ m$$



$3.2 - 0.109 - 0.0734 = 3.017$

$$\cos\theta = \frac{3.017}{3.2}$$

$\cos\theta = 0.943$

$\theta = 19.4°$ (3sf)

2 A particle P of mass 3.5 kg is moving down a line of greatest slope of a rough inclined plane. At the instant that its speed is $2.1\,\text{m s}^{-1}$ P is at a point A on the plane. At that instant an impulse of magnitude 33.6 N s, directed up the line of greatest slope, acts on P.

(i) Show that as a result of the impulse P starts moving up the plane with a speed of $7.5\,\text{m s}^{-1}$. **[2]**

While still moving up the plane, P has speed $1.5\,\text{m s}^{-1}$ at a point B where $AB = 4.2$ m. The plane is inclined at an angle of 20° to the horizontal. The frictional force exerted by the plane on P is modelled as constant.

(ii) Calculate the work done against friction as P moves from A to B. **[4]**

Impulse $= m\Delta v$

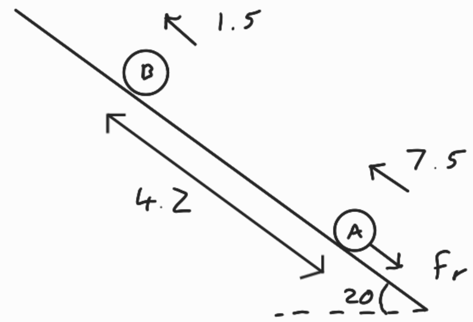
i. $I = m(v-u)$

$33.6 = 3.5(v - -2.1)$

$v + 2.1 = 9.6$

$v = 7.5\,\text{ms}^{-1}$

ii.



$s = 4.2$

$u = 7.5$

$v = 1.5$

$a = a$

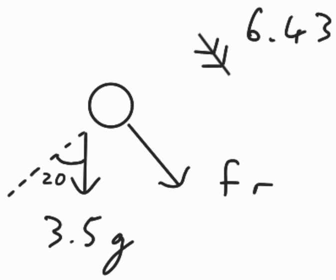
$t =$

$v^2 = u^2 + 2as$

$1.5^2 = 7.5^2 + 2a(4.2)$

$a = \dfrac{1.5^2 - 7.5^2}{2 \times 4.2}$

$a = -6.43 \text{ ms}^{-2}$



$f = ma$

$f_r + 3.5g \sin 20 = 3.5 \times 6.43$

$f_r = 10.769$

Work done $= 10.769 \times 4.2$

$= \underline{45.2 \text{ J}}$

Another method is possible :

KE at start $= \frac{1}{2} m v^2$

$= \frac{1}{2} \times 3.5 \times 7.5^2$

$= 98.4375 \text{ J}$

KE at end $= \frac{1}{2} m v^2$

$= \frac{1}{2} \times 3.5 \times 1.5^2$

$= 3.9375 \text{ J}$

Change in PE $= mgh$

$= 3.5 \times 9.8 \times 4.2 \sin 20$

$= 49.27 \text{ J}$

Work done by friction = loss of energy

$= 98.4375 - 3.9375 - 49.27$

$= 45.2 \text{ J}$

(iii) Hence find the magnitude of the frictional force acting on P. **[2]**

P first comes to instantaneous rest at point C on the plane.

(iv) Calculate AC. **[3]**

iii. Already known it used first method for ii.

Work done $= f \times d$

$45.2 = f \times 4.2$

$f = 10.8 \text{ N}$

iv. $s = s$

$u = 7.5$

$v = 0$

$a = -6.43$

$t =$

$v^2 = u^2 + 2as$

$0 = 7.5^2 + 2 \times -6.43 s$

$s = \frac{7.5^2}{2 \times 6.43}$

$s = AC = 4.4 \text{ m}$

3 A particle moves in a straight line with constant acceleration. Its initial and final velocities are u and v respectively and at time t its displacement from its starting position is s. An equation connecting these quantities is $s = k(u^\alpha + v^\beta)t^\gamma$, where k is a dimensionless constant.

(i) Use dimensional analysis to find the values of α, β and γ. **[6]**

$$s = k(u^\alpha + v^\beta)t^\gamma$$

$$L = \left[\left(\frac{L}{T}\right)^\alpha + \left(\frac{L}{T}\right)^\beta\right] \times T^\gamma$$

$$L = L^\alpha T^{\gamma-\alpha} + L^\beta T^{\gamma-\beta}$$

From the RHS, both parts need to have the same dimensions as the left.

$\Rightarrow L = \alpha \qquad 0 = \gamma - \alpha$

$\gamma = \alpha$

$\gamma = 1$

$\beta = 1$

$\therefore \underline{\alpha = \beta = \gamma = 1}$

(ii) By considering the case where the acceleration is zero, determine the value of k. **[2]**

If $a = 0$, $u = v$

$s = k(u+v)t$

$s = 2kut$

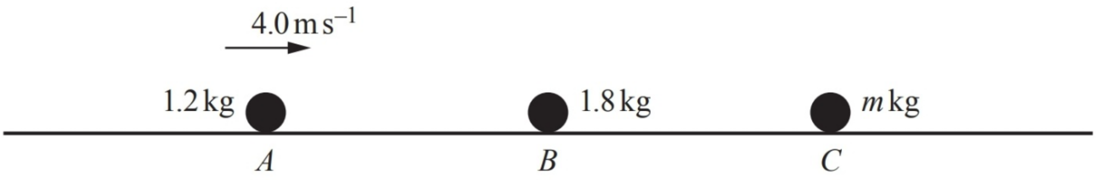
We know that speed $= \frac{\text{distance}}{\text{time}} \Rightarrow s = ut$

$\therefore \cancel{\mu}\cancel{t} = 2k\cancel{\mu}\cancel{t}$

$1 = 2k$

$k = 1/2$

4



Three particles A, B and C are free to move in the same straight line on a large smooth horizontal surface. Their masses are 1.2 kg, 1.8 kg and m kg respectively (see diagram). The coefficient of restitution in collisions between any two of them is $\frac{3}{4}$. Initially, B and C are at rest and A is moving with a velocity of $4.0\,\mathrm{m\,s^{-1}}$ towards B.

(i) Show that immediately after the collision between A and B the speed of B is $2.8\,\mathrm{m\,s^{-1}}$. **[4]**

(ii) Find the velocity of A immediately after this collision. **[1]**

i. Before: $A \rightarrow 4$ (1.2) $\quad B$ (1.8) $\quad C$ (m)

After: $v \leftarrow A$ (1.2) $\quad B \rightarrow w$ (1.8) $\quad C$ (m)

$$\frac{w - -v}{4} = e = \frac{3}{4}$$

$$v + w = 3$$

$$v = 3 - w$$

Momentum: $4(1.2) = 1.8w - 1.2v$

$$4.8 = 1.8w - 1.2(3 - w)$$

$$4.8 = 1.8w - 3.6 + 1.2w$$

$$8.4 = 3w$$

$$V_B = w = 2.8\,\mathrm{ms^{-1}}$$

ii. $v = 3 - w = 3 - 2.8 = 0.2$ (away from B)

B subsequently collides with *C*.

(iii) Find, in terms of *m*, the velocity of *B* after its collision with *C*. **[4]**

Before: $0.2 \leftarrow A$ (1.2) $\quad B \rightarrow 2.8$ (1.8) $\quad C$ (m)

After: $0.2 \leftarrow A$ (1.2) $\quad B \rightarrow P$ (1.8) $\quad C \rightarrow R$ (m)

$$\frac{R - P}{2.8} = e = \frac{3}{4}$$

$$R - P = 2.1$$
$$R = 2.1 + P$$

Momentum: $2.8 \times 1.8 = Rm + 1.8P$

$$5.04 = (2.1 + P)m + 1.8P$$
$$5.04 = 2.1m + Pm + 1.8P$$
$$P = \frac{5.04 - 2.1m}{m + 1.8}$$

(iv) Given that the direction of motion of *B* is reversed by the collision with *C*, find the range of possible values of *m*. **[2]**

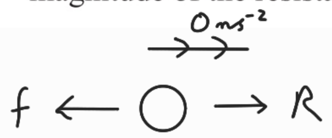
$P < 0$ if it has changed direction:

$$\frac{5.04 - 2.1m}{m + 1.8} < 0$$
$$5.04 - 2.1m < 0$$
$$5.04 < 2.1m$$
$$m > 2.4$$

5 The engine of a car of mass 1200 kg produces a maximum power of 40 kW.

In an initial model of the motion of the car the total resistance to motion is assumed to be constant.

(i) Given that the greatest steady speed of the car on a straight horizontal road is $42\,\mathrm{m\,s^{-1}}$, find the magnitude of the resistance force. **[2]**



f = constant resistance force,

$a = 0$ as constant speed

Power = force × speed

$40000 = R \times 42$

$R = 952.38$

as $a = 0$, $R - f = 0$

$\therefore$ $f = 952\,N$ (3sf)

The car is attached to a trailer of mass 200 kg by a light rigid horizontal tow bar. The greatest steady speed of the car and trailer on the road is now $30\,\mathrm{m\,s^{-1}}$. The resistance to motion of the trailer may also be assumed constant.

(ii) Find the magnitude of the resistance force on the trailer. **[2]**

$f_1 = 952.38\,N$ from i.

f_2 = resistive force acting on the trailer

Power = force × speed

$40000 = R \times 30$

$R = 1333.33$

$R - f_1 - f_2 = 0$

$f_2 = R - f_1$

$= 1333.33 - 952.38$

$= 381\,N$ (3sf)

The car and trailer again travel along the road. At one instant their speed is $15\,\mathrm{m\,s^{-1}}$ and their acceleration is $0.57\,\mathrm{m\,s^{-2}}$.

(iii) **(a)** Find the power of the engine of the car at this instant. **[4]**

(b) Find the magnitude of the tension in the tow bar at this instant. **[2]**

a. $R - f_1 - f_2 = ma$

$R - 952.38 - 380.95 = (1200 + 200) \times 0.57$

$R = 798 + 952.38 + 380.95$

$R = 2131.33$

Power $= R \times$ speed

$= 2131.33 \times 15$

$= 31969.95\ \mathrm{W}$

$= 32\ \mathrm{kW}$

b. Trailer:

$0.57 \twoheadrightarrow$

$380.95 \leftarrow \bigcirc \rightarrow T$

$200g$

Resultant Force $= ma$

$T - 380.95 = 200 \times 0.57$

$T = 114 + 380.95$

$T = 495\ \mathrm{N}$

In a refined model of the motion of the car and trailer the resistance to the motion of each is assumed to be zero until they reach a speed of $10\,\text{m}\,\text{s}^{-1}$. When the speed is $10\,\text{m}\,\text{s}^{-1}$ or above the same constant resistance forces as in the first model are assumed to apply to each.

The car and trailer start at rest on the road and accelerate, using maximum power.

(iv) Without carrying out any further calculations,

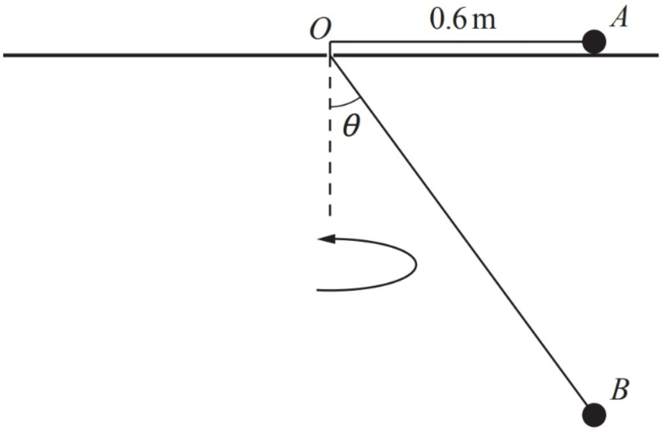
(a) explain whether the time taken to attain a speed of $20\,\text{m}\,\text{s}^{-1}$ would be predicted to be lower, the same or higher using the refined model compared with the original model, **[2]**

(b) explain whether the greatest steady speed of the system would be predicted to be lower, the same or higher using the refined model compared with the original model. **[2]**

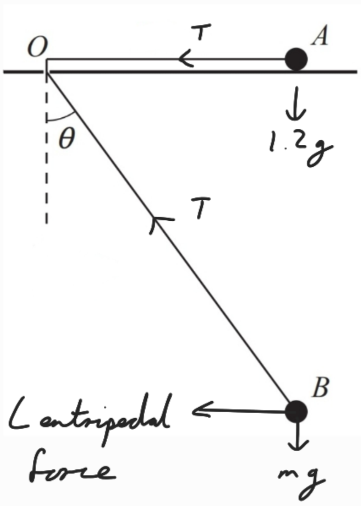
a. Time taken will be lower because acceleration will be greater (when below $10\,ms^{-1}$) due to the lack of resistance forces (so acceleration will be greater).

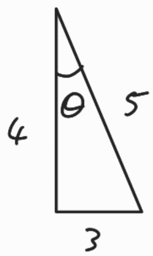
b. Maximum speed will be the same as this depends only on the resistance acting on the system at high speeds.

6 Two particles A and B are connected by a light inextensible string. Particle A has mass 1.2 kg and moves on a smooth horizontal table in a circular path of radius 0.6 m and centre O. The string passes through a small smooth hole at O. Particle B moves in a horizontal circle in such a way that it is always vertically below A. The angle that the portion of the string below the table makes with the downwards vertical through O is θ, where $\cos\theta = \frac{4}{5}$ (see diagram).



(i) Find the time taken for the particles to perform a complete revolution. **[7]**





$\cos\theta = 4/5$

$\sin\theta = 3/5$

For B: Resolving vertically:

$$mg = T\cos\theta$$

$$mg = 4T/5$$

$$T = \frac{5mg}{4}$$

Resolving horizontally: $T\sin\theta = m \times 0.6 \times w^2$

$$\frac{3T}{5} = 0.6mw^2$$

$$\frac{3}{5}\left(\frac{5\cancel{m}g}{4}\right) = 0.6\cancel{m}w^2$$

$$\Rightarrow \quad \frac{3g}{4} = 0.6\, w^2$$

$$w^2 = 12.25$$

$$w = 3.5 \text{ rad s}^{-1}$$

$\therefore$ Time $= \dfrac{2\pi}{w} = \dfrac{2\pi}{3.5} = \underline{1.8 \text{ sec}}$ per revolution

(ii) Find the mass of B. **[3]**

Resolve horizontally for A: $T = 1.2 \times 0.6 \times w^2$

$$T = 8.82$$

from i.: $T = \dfrac{5mg}{4}$

$$\underline{m = 0.72} \text{ kg}$$